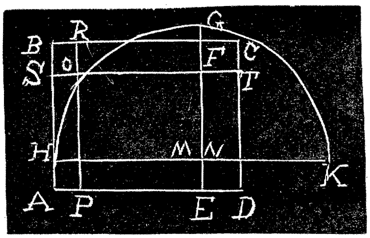
387. Proposed by DANIEL KRETH, Oxford, Iowa.

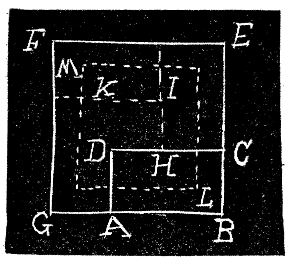
A lot 100 feet long and 60 feet wide, has a walk extending from one corner half way around it, and occupying one-third of the area. Required the width of the walk. A geometrical construction is desired.

I. Solution by A. H. HOLMES, Brunswick, Maine.



Let $ABCD$ be the lot. $AB=CD=60$, and $AD=BC=100$. On DA take DE=one third of CD, and draw EF parallel to CD and cutting BC in F. On EF take EN=one fourth of DE and draw HN parallel to AD, the point H being on AB and $AH=EN$. On HN extended take $NK=NF$. Bisect HK in M, and with MH as radius describe a semi-circle on HK. Extend EF to cut the circumference in G. On EA take $EP=NG$. Draw PR parallel to AB. Take $BS=AP$, and draw ST parallel to BC. Then on ST take $SO=AP$. Then $ABRP+ORCT=\frac{1}{3}ABCD$, and AP =width of walk required; which is shown as follows: Let x=width of walk. Then $100x+(60-x)x=2000$. Therefore, $(80-x)^2=55\times80$.

II. Solution by C. E. GITHENS, Ph. D., Wheeling, West Virginia.



Let $ABCD$ be the given lot. Form the square $GBEF$ by arranging three other equal lots as in the figure. Then $GB=60$ feet+100 feet =160 feet.

Area of square $DHIK=40^2$ square feet

Area of square $LM=\frac{2}{3}(4ABCD)+DHIK=$ 17,600 square feet.

Hence, side of square $LM=\sqrt{(17,600)}$ feet $=40\sqrt{11}$ feet.

Hence, width of walk$=\frac{1}{2}(160-40\sqrt{11})$ feet$=(80-20\sqrt{11})$ feet.

Also solved by H. Prime and S. G. Barton.

388. Proposed by WILLIAM HOOVER, Ph. D., Professor of Mathematics and Astronomy, Ohio University Athens, Ohio.

A conic is inscribed in a triangle and one focus lies on the polar circle of the triangle. Prove that the corresponding directrix passes through the center of perpendiculars.

Solution by the PROPOSER.

Reciprocating with respect to the focus, the conic corresponds to the circumscribing circle of the reciprocal triangle; the polar circle, whose center is the orthocenter of the fixed triangle, to a parabola with focus, the fixed focus of the given conic, the given orthocenter to the directrix of the

reciprocal parabola, the directrix of the conic to the center of the reciprocal circle which is on the directrix of the parabola.

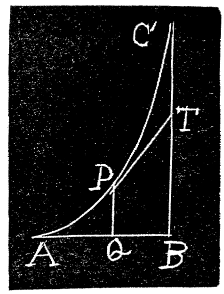
CALCULUS.

314. Proposed by REV. J. H. MEYER, S. J., New Orleans, La.

A fox started from a certain point and ran due east 300 yards, when it was overtaken by a hound that started from a point 100 yards due north of the fox's starting point, and ran directly towards the fox throughout the race. Find the lengh of the curve described by hound, both having started at the same instant, with a uniform velocity.

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland, and FRANCIS E. RUST, E. E., Pittsburg, Pa.

Let A be the starting point of the hound, and B that of the fox, C the point of capture, P some point of the curve described by the hound, $AQ=x$, $PQ=y$, DPT a tangent to the curve at P, $AB=a$ $(=100)$, $BC=b(=300)$; $AP=s$, m=rate of hound, n= rate of fox; $s=mt$, $BT=nt$, t being a certain time. $BT=y+(a-x)\tan TBE=y+(a-x)\dfrac{dy}{dx}$.



$\therefore \dfrac{s}{m}=\dfrac{y+(a-x)(dy/dx)}{n}$; or, putting $\dfrac{n}{m}=\beta$, $\beta s=y+(a-x)\dfrac{dy}{dx}$. Differentiating, $\beta\dfrac{ds}{dx}=(a-x)\dfrac{d^2y}{dx^2}$; or

$\beta\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=(a-x)\dfrac{d^2y}{dx^2}$. Putting $\dfrac{dy}{dx}=p$, $\beta\sqrt{(1+p^2)}=(a-x)\dfrac{dp}{dx}$.

$\therefore \beta\dfrac{dx}{a-x}=\dfrac{dp}{\sqrt{(1+p^2)}}$. $\therefore \dfrac{C_1}{(a-x)^\beta}=p+\sqrt{(1+p^2)}$, and since for $x=0$, $p=0$, $\dfrac{C_1}{a^\beta}=1$, and $C_1=a^\beta$. Now, $\dfrac{a^\beta}{(a-x)^\beta}=p+\sqrt{(1+p^2)}$, or $a^\beta(a-x)^{-\beta}=p+\sqrt{(1+p^2)}$; whence $p=\dfrac{dy}{dx}=\frac{1}{2}[a^\beta(a-x)^{-\beta}-a^{-\beta}(a-x)^\beta]$.

$\therefore y=\dfrac{a^{-\beta}(a-x)^{1+\beta}}{1+\beta}-\dfrac{a^\beta(a-x)^{1-\beta}}{1-\beta}+C_2$; but $0=\frac{1}{2}\left(\dfrac{a^{-\beta}a^{1+\beta}}{1+\beta}-\dfrac{a^\beta a^{1-\beta}}{1-\beta}\right)+C_2$, whence $C_2=\dfrac{a^\beta}{1-\beta^2}$.

$\therefore y=\frac{1}{2}\left[\dfrac{a^{-\beta}(a-x)^{1+\beta}}{1+\beta}-\dfrac{a^\beta(a-x)^{1-\beta}}{1-\beta}\right]+\dfrac{a^\beta}{1-\beta^2}$; $\dfrac{ds}{dx}=\frac{1}{2}[a^\beta(a-x)^{-\beta}+a^{-\beta}(a-x)^\beta]$;